EFFECT OF ECCENTRICITY ON THE PUMPING CAPACITY IN AN UNBAFFLED VESSEL

A. Hidalgo-Millán,[1] E. Soto,[1] R. Zenit[2] and G. Ascanio[1]*

1. *Centro de Ciencias Aplicadas y Desarrollo Tecnológico, Universidad Nacional Autónoma de México, Ciudad Universitaria, 04510 DF, Mexico*
2. *Instituto de Investigaciones en Materiales, Universidad Nacional Autónoma de México, Ciudad Universitaria, 04510 DF, Mexico*

The aim of this study was to investigate the effect of the shaft eccentricity on the pumping capacity in stirred vessels estimated with the aid of the particle image velocimetry technique with impellers in centred and off-centred positions. For the off-centred case, 19 different planes in the azimuthal direction were taken to obtain a reliable average velocity into the whole tank. The velocity and vorticity fields were evaluated for all positions. A Rushton turbine and a pitched blade turbine (PBT) were operated in laminar and turbulent regimes. The pumping number increased considerably with eccentricity for the Rushton turbine (radial flow), whereas the effect for the PBT turbine (axial flow) was almost negligible.

Keywords: stirred vessels, eccentric impellers, pumping number

INTRODUCTION

Mixing in stirred vessels at low-to-moderate Reynolds number (*Re*) is usually inefficient due to the flow structures generated at the onset of operation. Bresler et al. (1997) noticed the presence of isolated regions when using open impellers operating in the laminar regime. Pseudocaverns, defined as well-mixed regions surrounding the impeller are generated when a Newtonian fluid is stirred. Furthermore, dead zones can be formed far away from the impeller. As a consequence, the mixing time required to reach a desirable homogeneity level is very long or infinite, resulting in an expensive and inefficient process. Such flow structures can be removed or avoided by increasing the impeller speed. Since the energy consumption increases also with the impeller speed, other approaches should be considered (Franjione et al., 1989). The use of dynamic perturbations by means of unsteady rotational speed was proposed by Yao et al. (1998), who demonstrated that mixing times are significantly reduced. Other approach is based on the use of geometrical perturbations by displacing the agitation shaft from the vessel centreline. Alvarez et al. (2002); and Ascanio et al. (2002) found that flow structures (pseudocaverns) are readily destroyed when using eccentric impellers; as a consequence shorter mixing times are obtained. Geometrical and dynamic perturbations are based on the seminal works of Aref (1984), who demonstrated that chaotic flow prevents the formation of coherent regions in the vicinity of off-centred cylinders. Aref and Balachandar (1986) confirmed such findings. The hydrodynamic characteristics of stirred vessels equipped with off-centred impellers in the turbulent regime have also reported elsewhere. Montante et al. (2006) found that geometrical perturbations induce complex features to flow patterns in unbaffled stirred vessels. Recently, Galletti and Brunazzi (2008); and Galleti et al. (2009) observed the presence of two vortical structures, one above and one below the impeller with eccentrically placed impeller even in the turbulent regime.

Scarce information on the hydrodynamics in stirred unbaffled vessels with off-centred impellers in the laminar regime is available in the literature. Dimensionless analysis is commonly used to characterise the pumping ability of an impeller in stirred vessels, which directly impacts on the process efficiency. Aubin et al. (2001) compared the pumping capacity of a six-blade pitched blade turbine (PBT) with a Mixel TT (MTT) impeller, which is a hydrofoil-type three-blade axial-flow impeller. Although, the PBT has six blades, they found that the MTT is more effective

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*Author to whom correspondence may be addressed.
E-mail address: gabriel.ascanio@ccadet.unam.mx

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in the down-pumping mode. Zadghaffari et al. (2009) carried out a numerical study on the hydrodynamics of a double-Rushton turbine array, whose results were experimentally compared in terms of the pumping capacity. For this particular case, they found that the pumping capacity in terms of the pumping number (N_Q) varies slightly with the impeller velocity. Rice et al. (2006) performed an experimental study in the laminar flow regime with a Rushton turbine in a stirred vessel at three different *Re* (1, 10, and 28) to evaluate the pumping capacity. They reported a reciprocating motion with negligible net pumping as the *Re* decreases to its minimum value. The use of eccentric impellers have also been studied in turbulent regime by Montante et al. (2006), who performed a numerical and experimental study. They found that care must be exerted when choosing the turbulent models with unbaffled stirred vessels. Hall et al. (2005) reported the effect of impeller eccentricity on the turbulent kinetic energy and the flow fields. However, to our knowledge, a quantitative evaluation of the relation between eccentricity and pumping capacity has not been reported to date. In this work, the hydrodynamics of an unbaffled vessel stirred with a Rushton turbine and PBT located in both centred and off-centred positions was characterised by flow visualisation by means of the particle image velocimetry (PIV). An alternative method to calculate the pumping number under steady and unsteady conditions in laminar and transitional regimes has been developed.

MATERIALS AND METHODS

Aqueous solutions of polyethylene glycol at 10 (wt.%) and pure glycerol, whose properties are shown in Table 1, were used as Newtonian fluids. The dynamic viscosity of the fluids was determined with a rheometer AR-2000 (TA-Instruments). Figure 1a shows the experimental setup, which consists of a polycarbonate transparent cylindrical vessel of a diameter (T) of 165 mm and a liquid height $H = T$. In order to avoid optical disruption, the vessel was placed inside a square tank filled with the same fluid under study. The shaft was driven by a DC motor of 248 W, with a controller in an open-loop mode. A radial flow discharge impeller (Rushton turbine) and an axial flow discharge impeller (45° PBT), both with a diameter of 55 mm ($D = T/3$), were tested. The shape of the Rushton turbine blades was square with 5 mm by side; the PBT impeller had 4-square blades of 20 mm at 45° (Figure 1d). The clearance between the impeller and the bottom tank was $T/3$, which is equivalent to 55 mm. Two different Reynolds numbers ($Re = \rho N D^2/\mu$) corresponding to the laminar and transition regimes were studied. Under these two conditions, the superficial vortex did not generate significant perturbations such as incorporation of air into the liquid. Air bubbles were not observed to be captured by the impeller vortex and the velocity uncertainty was negligible. Experiments were conducted at room temperature and the duration of experiments was less than 3 min in order to avoid a significant change of temperature due to viscous dissipation. Table 1 shows the fluid properties and impeller velocity for every condition.

Table 1. Liquid properties and impeller velocity

Fluid	Density, ρ (kg/m^3)	Speed, N (s^{-1})	Viscosity, μ (Pa.s)	*Re*
PEG 10 (wt.%)	1,010	5.9	0.02	901
Glycerol	1,261	2.3	1.1	8

The impeller eccentricity was defined by $X^* = x/r$, where x is the distance from the vessel centreline to the impeller radial position ($x = 30$ mm) and r is the vessel radius ($T/2$). A PIV system by Dantec Dynamics was used to obtain the two-dimensional (2D) flow patterns (Figure 1a). The stirred vessel was illuminated with a Nd:YAG laser (wavelength of 532 nm) providing a light sheet of the order of 1 mm width. The liquid was seeded with silver-coated hollow glass particles with a mean diameter of 10 μm and an average density of 1.4 g/cm^3. The particles properties and the viscosity of the liquids guaranteed that the sedimentation time of the particles was negligible in comparison with the fluid velocity. To verify that the particles remained suspended in the fluid, a test was done afterwards to ensure that the particles remained well distributed in the tank. The equipment was turned off during a minimum of 6 h, and then was illuminated with the laser in order to corroborate the particle distribution. The effect of the density differences between the particles and the fluid caused by inactivity were not significant on the particle distribution.

The objective of this work was to estimate the pumping capacity of eccentric impellers at different angular positions. Due to scarce information about the estimation of the pumping capacity in mixing systems using eccentric impellers, 9 different measurements of the velocity field in the θ direction every 10° were performed by taking one blade as reference for the 0° position. These positions were studied to obtain the variation of velocities that are present in a mixing system with an eccentric impeller, due to the interaction between it and the tank wall. For this purpose, a mechanical device was designed. It consisted of a base on a circular track that supports the tank, so that the tank rotated at any desired angular position (Figure 1a–c).

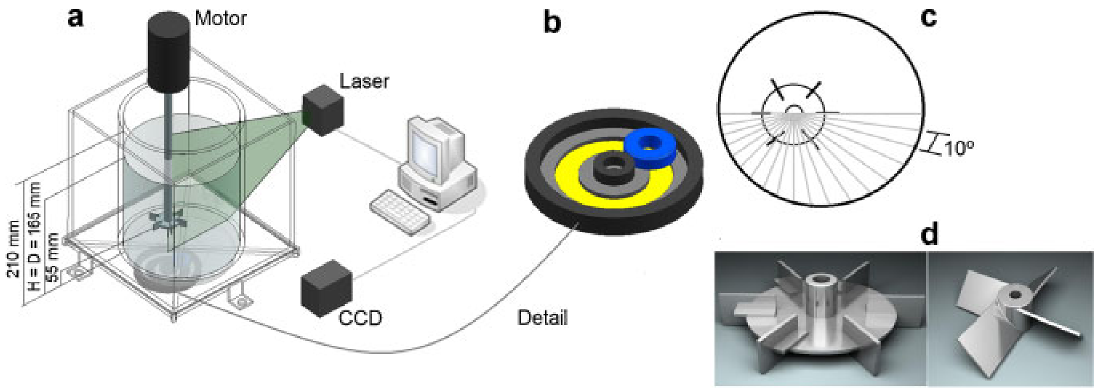

Figure 1. Experimental setup. (a) Vessel, PIV arrangement and support for switching the PIV planes. (b) Detail of the circular track. (c) Top view of the 19 positions of the PIV planes. (d) PBT and Rushton impellers.

In each position, 600 measurements were taken and averaged in order to obtain a reliable pumping capacity estimation. The time between frames was calculated to obtain images every 5°, such that a reliable swept of the fluid velocity on the tip impeller boundaries could be obtained. The advantage of this arrangement is that PIV system is placed in one position only, hence calibration was the same for all the measurements.

RESULTS AND DISCUSSION

Flow Patterns

The flow around the centred impeller for the laminar flow (pure glycerine) was characterised by the formation of ordered structures. In the case of Rushton turbine, the liquid is ejected in the radial direction towards the vessel walls and generates two

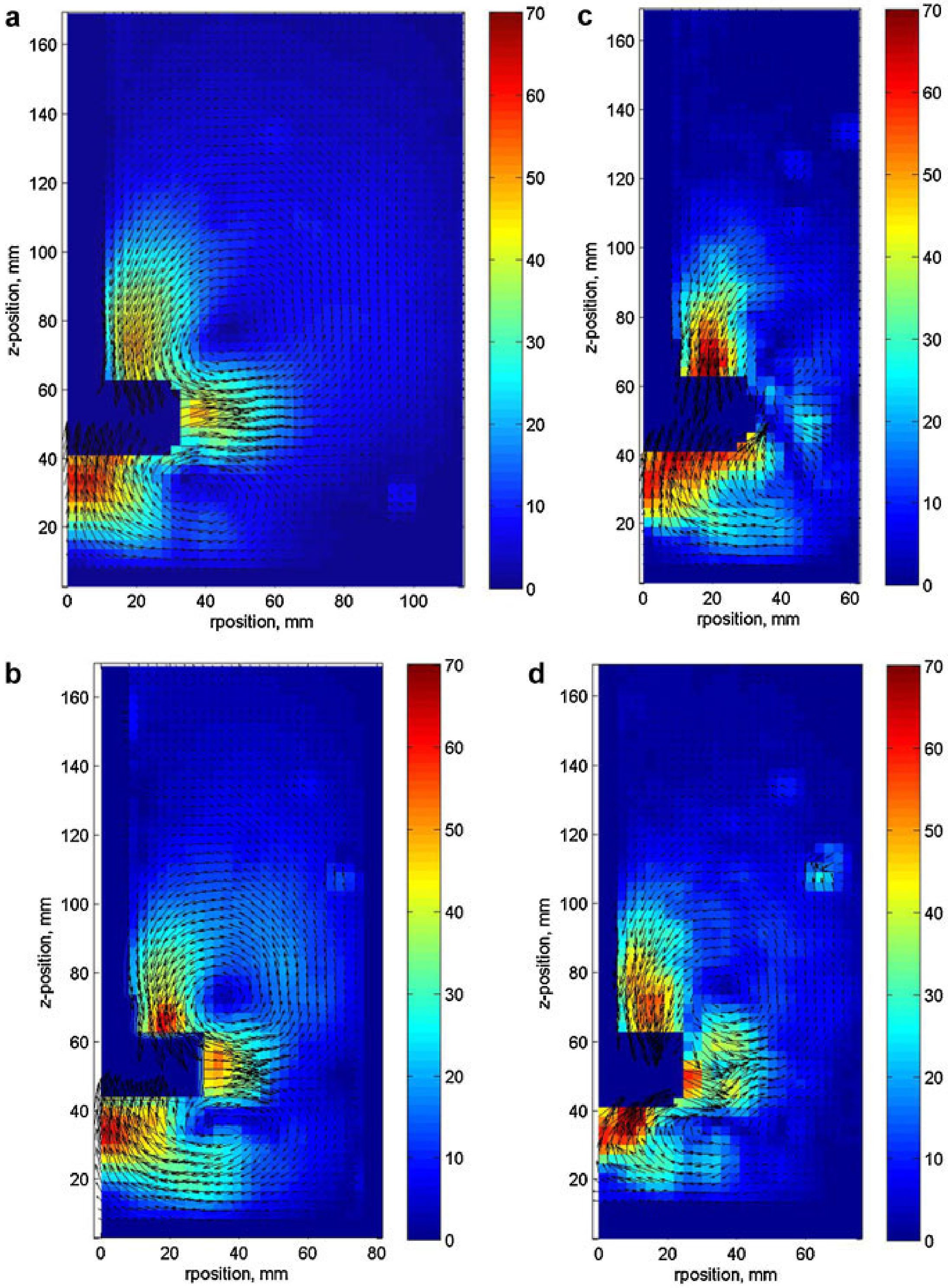

Figure 2. Velocity fields and mean velocity generated by a Rushton turbine at $Re = 8$: (a) 0° (bar scale in mm/s); (b) 90° (bar scale in mm/s); (c) 180° (bar scale in mm/s); and (d) centred (bar scale in mm/s).

toroidal structures, one above and another one below the impeller. A 2D view is presented in Figure 2 (right). For the PBT case, liquid is ejected in the axial direction and only one single vortex is generated (Figure 3d). As the impeller is off-centred, the flow was constrained towards the closer wall and becomes asymmetric, but the general behaviour is similar to that of the centred position, which can be observed in Figures 2 and 3. For the centred impeller, due to the axial symmetry, only half the tank is visualised. However, for the off-centred impeller position, 19 different orientations were measured every 10°. In Figure 2 (Rushton turbine), and Figure 3 (45° PBT), only 3 out of the set of 19 (0°, 90°, and 180°) are presented. The images show the velocity fields (vectors), and the fluid velocity magnitude (colours) with pure glycerol. Furthermore, the velocity values reached by using the PBT are larger by

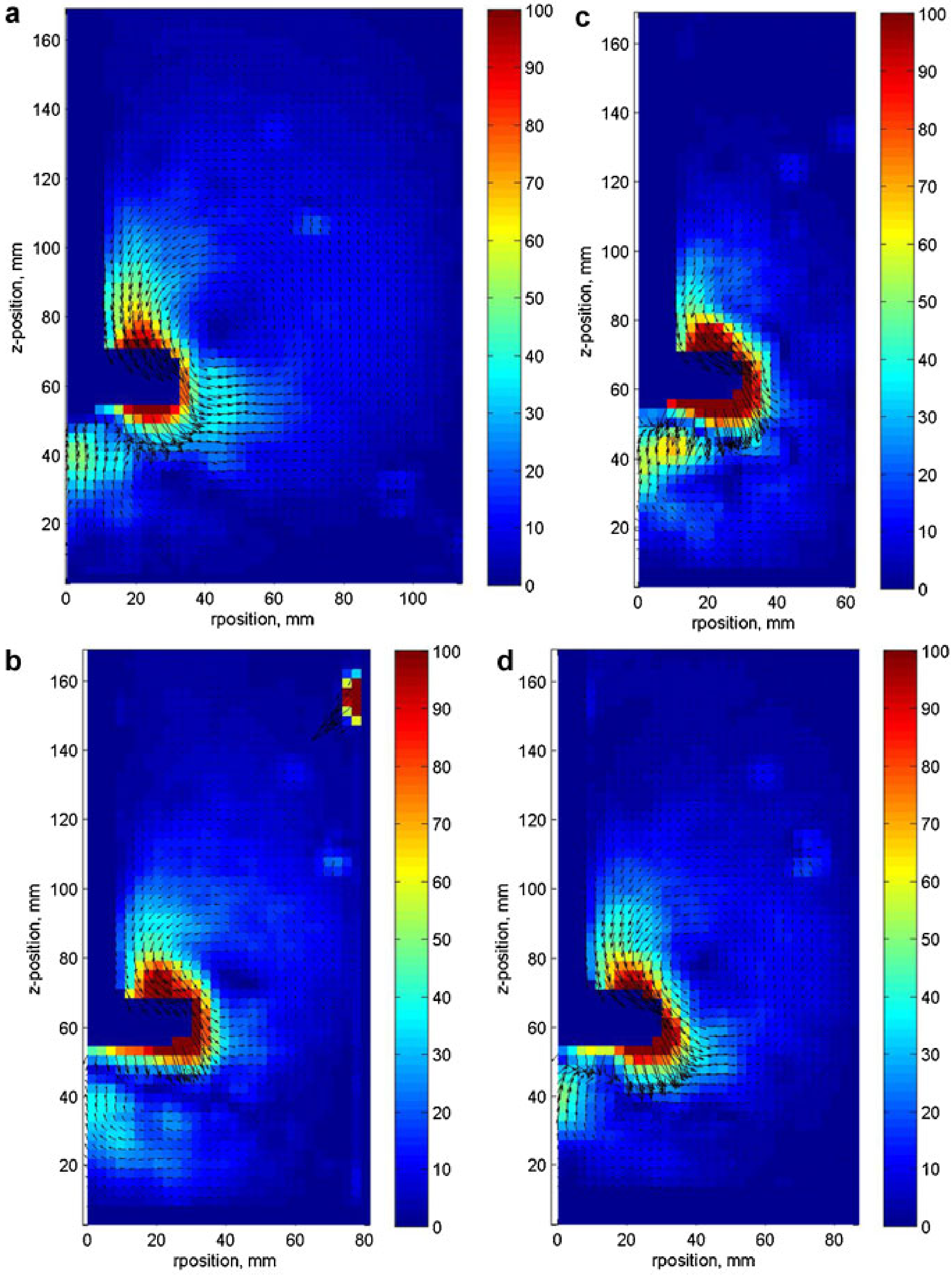

Figure 3. Velocity fields and mean velocity generated by a PBT at $Re = 8$: (a) 0° (bar scale in mm/s); (b) 90° (bar scale in mm/s); (c) 180° (bar scale in mm/s); and (d) centred (bar scale in mm/s).

up to 30% than those reached by the Rushton turbine. Under this flow regime (laminar), fluid motion is observed around impellers while in remote areas the fluid is nearly stagnant.

For the case of aqueous solutions of polyethylene glycol, *Re* was fixed to 901, which corresponds to the transitional regime. As consequence, the momentum transference is increased and mixing is improved. This fact can be explained from the extent of fluid circulation to a higher volume of the tank. The flow patterns for both impellers are shown in Figures 4 and 5. In this case, the fluid velocity magnitude was larger for the Rushton turbine compared to that produced by the PBT; it seems that the interaction with the wall is more relevant for radial discharge than axial discharge.

Pumping Capacity

The benefits of using eccentric impellers under laminar and transitional regimes have been well-recognised and reported elsewhere (Alvarez et al., 2002; Ascanio et al., 2002; Karcz et al., 2005; Montante et al., 2006). Also, it is well know that mixing is improved by geometric perturbations, such as the impeller

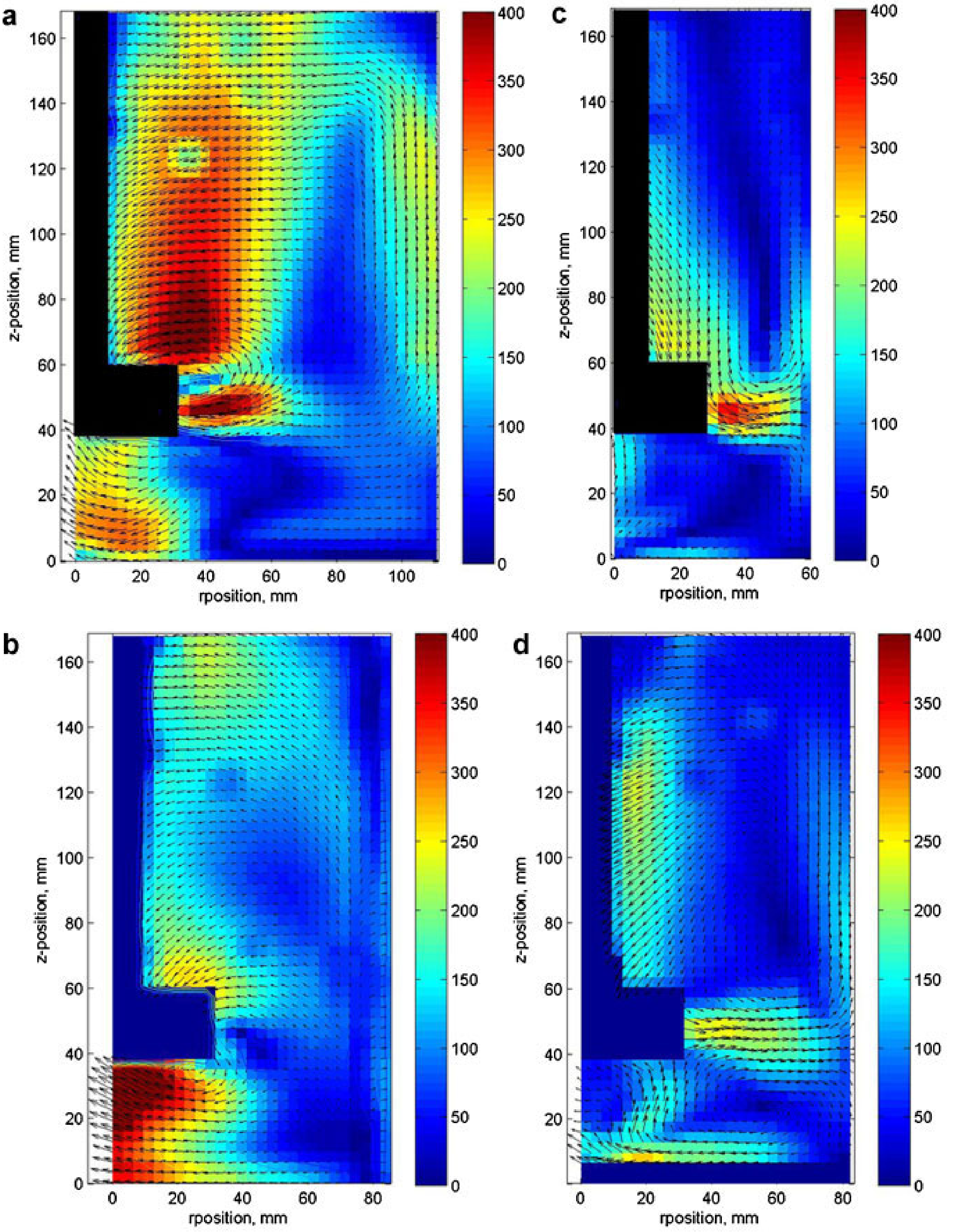

Figure 4. Velocity fields and mean velocity generated by a Rushton turbine at *Re* = 901: (a) 0° (bar scale in mm/s); (b) 90° (bar scale in mm/s); (c) 180° (bar scale in mm/s); and (d) centred (bar scale in mm/s).

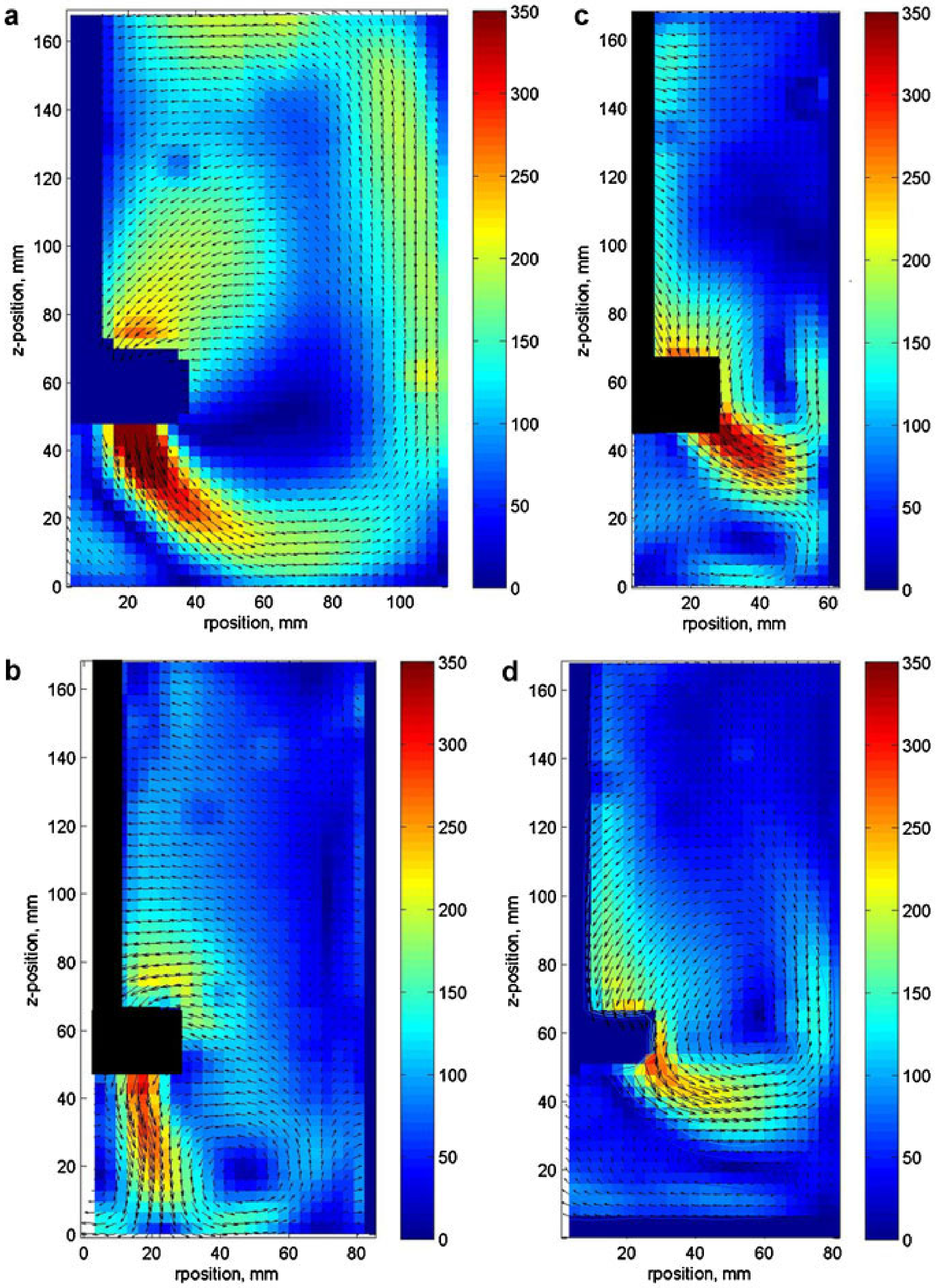

Figure 5. Velocity fields and mean velocity generated by a PBT at $Re = 901$: (a) 0° (bar scale in mm/s); (b) 90° (bar scale in mm/s); (c) 180° (bar scale in mm/s); and (d) centred (bar scale in mm/s).

eccentricity (Alvarez et al., 2002; Ascanio et al., 2002). Most of the authors explain this behaviour from an increment of the pumping capacity. However, in the literature there is not a reliable measurement of this quantity for eccentric impeller positions. In the present work, a methodology for this purpose was presented. The circulation flow, Q, is calculated from the average of the 19 different positions previously mentioned. The pumping capacity is defined by using the pumping number (Equation 1):

$$N_Q = \frac{Q}{ND^3} \tag{1}$$

where N is the rotational speed (rev/s), D is the impeller diameter (m) and Q is the circulation flow eaving the turbine blades, defined by:

$$Q = \int_{z^{--}}^{z^{++}} \pi D \left| \left(v_r^0\right)_{r=r^{++}} \right| \mathrm{d}z + \int_{0}^{r^{++}} 2\pi r \left| \left(v_z^0\right)_{z=z^{++}} \right| \mathrm{d}r + \int_{z^{--}}^{z^{++}} 2\pi r \left| \left(v_z^0\right)_{z=z^{--}} \right| \mathrm{d}r \qquad (2)$$

where, z^{++}, z^{--} and r^{++} are the virtual boundaries of the control volume around the impeller, placed 2 mm from impeller tip. The superscript '0' refers to the fluid moving out of this control volume. This expression takes into account both radial and axial pumping flow rates (Aubin et al., 2001). The fluid velocity v_z and v_r are the average value of 600 measurements, which guarantee the statistical convergence of the measurements. The circulation flow (Equation 2) was calculated using the PIV data, solving the discrete integral as:

$$Q = \sum_{i=1}^{n} V_i A_i \qquad (3)$$

where V_i is the fluid velocity '*i*' on the vector map estimated by the PIV technique, and A_i is the cross-sectional area crossed by the fluid at its corresponding velocity. This equation was used in all the virtual boundaries of the control volume.

Table 2 summarises the pumping numbers obtained for the different conditions investigated. The pumping capacity of the Rushton turbine in the laminar regime is ingood agreement with the results obtained by Rice et al. (2006); and Xuereb et al. (2006). In the other cases, no information has been found to further verify our results.

In the case of a Rushton turbine, the pumping capacity estimated under off-centred conditions was almost twice than the one obtained with the centred impeller. However, the effect of eccentricity is very small for the PBT in both flow regimes. In addition, Figure 6 shows the dimensionless volumetric flow (flow in the eccentric case with respect to the centric case) for every angular position at the two different *Re* numbers. In the case of the PBT, the volumetric flow rate was affected slightly by the impeller eccentricity, however, the flow remained almost constant as the *Re* is increased. In the case of the Rushton turbine, the flow rate increased as the *Re* increased and the benefits of eccentricity are more evident. For the Rushton turbine, the volumetric flow increased at all the different angular position studied at $X^* = 0.36$. On the other hand, the flow reached lower values for the angular positions corresponding between 90° and 150°.

These results show that the pumping capacity is improved when using radial discharge impeller in the off-centred position; however, this phenomenon did not occur with axial impellers. This increase on the pumping capacity obtained with radial impellers can be the result of the interaction between the tank wall and the impeller tip blade. These perturbations did not occur in axial impellers due to the flow discharge direction, towards the tank bottom. This interaction generated an azimuthal disturbance, in other words, the flow becomes asymmetric. Flow lines are narrowed on the wall vicinity generating vorticity in this zone. Therefore, the vortices did not have an uniform strength in the angular direction, which make them unstable. This phenomenon generates velocity fluctuations and, as a result, the mixing is improved.

Table 2. Pumping number for different conditions

		Rushton turbine		PBT	
Fluid	*Re*	Off-centred	Centred	Off-centred	Centred
PEG solution 10% (w/w)	901	0.58	0.31	0.60	0.65
Glycerol	8	0.22	0.14	0.22	0.25

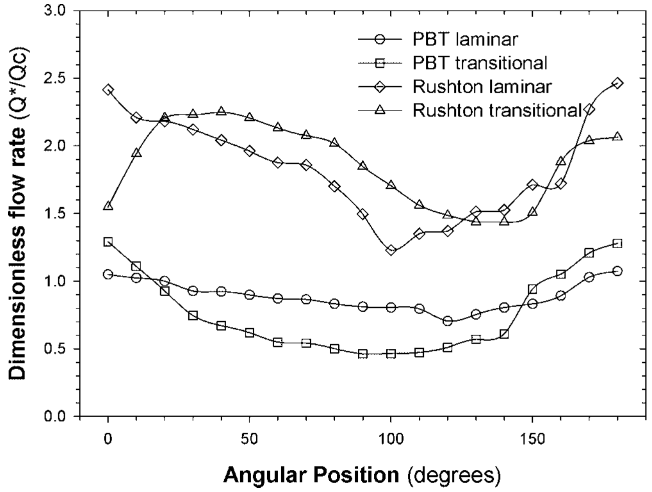

Figure 6. Dimensionless volumetric flow rate as a function of the angular position.

CONCLUSIONS

An alternative method to estimate the pumping capacity of eccentric impellers in stirred vessels has been developed. The proposal was based on the measurement of several planes at different angular positions around the impellers. Then the pumping capacity was estimated for every position and averaged. It has been demonstrated that pumping capacity in the laminar and transitional regimes was increased when using a radial discharge impeller (Rushton turbines) in off-centred position. These results suggest that the interaction of the flow with the wall increased the circulation of the fluid. However, a significant improvement was not observed with axial flow impeller (PBT), which is a consequence of the poor interaction of the flow with the vessel walls. To validate these results, it is necessary to conduct more experiments for different eccentricities with axial and radial impellers. It is also important to further investigate the implications of the increasing pumping capacity on the power consumption and mixing times.

NOMENCLATURE

A	cross-sectional area (m^2)
D	impeller diameter (mm)
T	tank diameter (mm)
H	height of liquid in the tank (mm)
N	impeller speed (1/s)
N_Q	pumping number
PBT	pitched blade turbine
PEG	polyethylene glycol
PIV	particle image velocimetry
Q	circulation flow (m^3/s)
Q_c	circulation flow in centred position (m^3/s)
Q^*	circulation flow in off-centred position (m^3/s)
r	vessel radius (mm)

Re	Reynolds number
r^{++}	imaginary border located 2 mm in front of impeller
v	average velocity fluid moving out of the control volume (m/s)
V	velocity (m/s)
x	distance from vessel centreline to the impeller radial position (mm)
X^*	impeller eccentricity
z^{++}	imaginary border located 2 mm above the impeller
z^{--}	imaginary border located 2 mm below the impeller

Greek Symbols

μ	fluid viscosity (Pa.s)
ρ	density (kg/m^3)

Subscripts

i	element
r	direction *r*
z	direction *z*

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REFERENCES

Alvarez, M., P. E. Arratia and F. J. Muzzio, “Laminar Mixing in Eccentric Stirred Tank Systems,” Can. J. Chem. Eng. **80**(4), 546–557 (2002).

Aref, H., “Stirring by Chaotic Advection,” J. Fluid Mech. **143**, 1–21 (1984).

Aref, H. and S. Balachandar, “Chaotic Advection in a Stokes Flow,” Phys. Fluids **29**(11), 3515–3521 (1986).

Ascanio, G., M. Brito-Bazan, E. Brito-de La Fuente, P. J. Carreau and P. A. Tanguy, “Unconventional Configuration Studies to Improve Mixing Times in Stirred Tanks,” Can. J. Chem. Eng. **80**(4), 558–565 (2002).

Aubin, J., P. Mavros, D. F. Fletcher, J. Bertrand and C. Xuereb, “Effect of Axial Agitator Configuration (Up-Pumping, Down-Pumpimg, Reverse Rotation) on Flow Patterns Generated in Stirred Vessels,” Trans. IChem. E **E 79**(Part A), 845–856 (2001).

Bresler, L., T. Shinbrot, G. Metcalfe and J. M. Ottino, “Isolated Mixing Regions: Origin, Robustness and Control,” Chem. Eng. Sci. **52**(10), 1623–1636 (1997).

Franjione, J. G., C. W. Leong and J. M. Ottino, “Symmetries Within Chaos: A Route to Effective Mixing,” Phys. Fluids **1**(11), 1772–1783 (1989).

Galleti, C., S. Pintus and E. Brunazzi, “Effect of Shaft Eccentricity and Impeller Blade Thickness on the Vortices Features in an Unbaffled Vessel,” Chem. Eng. Res. Des. **87**(4), 391–400 (2009).

Galletti, C. and E. Brunazzi, “On the Main Flow Features and Instabilities in an Unbaffled Vessel Agitated With an Eccentrically Located Impeller,” Chem. Eng. Sci. **63**(18), 4494–4505 (2008).

Hall, J. F., M. Barigou, M. J. H. Simmons and E. H. Stitt, “A PIV Study of Hydrodynamics in Gas–Liquid High Throughput Experimentation (HTE) Reactors With Eccentric Impeller Configurations,” Chem. Eng. Sci. **60**, 6403–6413 (2005).

Karcz, J., M. Cudak and J. Szoplik, “Stirring of a Liquid in a Stirred Tank With an Eccentrically Located Impeller,” Chem. Eng. Sci. **60**, 2369–2380 (2005).

Montante, G., A. Bakker, A. Paglianti and F. Magelli, “Effect of the Shaft Eccentricity on the Hydrodynamics of Unbaffled Stirred Tanks,” Chem. Eng. Sci. **61**, 2807–2814 (2006).

Rice, M., J. Hall, G. Papadakis and M. Yianneskis, “Investigation of Laminar Flow in a Stirred Vessel at Low Reynolds Numbers,” Chem. Eng. Sci. **61**, 2762–2770 (2006).

Xuereb, C., M. Poux and J. Bertrand, “Agitation et Mélange: Aspects Fondamentaux et Applications Industrielles,” Ed. Dunod, France (2006).

Yao, W. G., H. Sato, K. Takahashi and K. Koyama, “Mixing Performance Experiments in Impeller Stirred Tanks Subjected to Unsteady Rotational Speeds,” Chem. Eng. Sci. **53**, 3031–3040 (1998).

Zadghaffari, R., J. S. Moghaddas and J. Revstedt, “A Mixing Study in a Double-Rushton Stirred Tank,” Comp. Chem. Eng. **33**, 1240–1246 (2009).

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